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Grand unification in the projective plane

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Abstract: A 6-dimensional grand unified theory with the compact space having the topology of a real projective plane, i.e., a 2-sphere with opposite points identified, is considered. The space is locally flat except for two conical singularities where the curvature is concentrated. One supersymmetry is preserved in the effective 4d theory. The unified gauge symmetry, for example SU(5) , is broken only by the non-trivial global topology. In contrast to the Hosotani mechanism, no adjoint Wilson-line modulus associated with this breaking appears. Since, locally, SU(5) remains a good symmetry everywhere, no UV-sensitive threshold corrections arise and SU(5)-violating local operators are forbidden. Doublet-triplet splitting can be addressed in the context of a 6d $N = 2$ super Yang-Mills theory with gauge group SU(6). If this symmetry is first broken to SU(5) at a fixed point and then further reduced to the standard model group in the above non-local way, the two light Higgs doublets of the MSSM are predicted by the group-theoretical and geometrical structure of the model.

Keywords: Field Theories in Higher Dimensions, Extra Large Dimensions, Beyond Standard Model, GUT.

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Contents

1. Introduction

The mechanism of gauge symmetry breaking is an important open issue in the context of grand unified theories (GUTs). In the conventional approach, where the symmetry is broken by the vacuum expectation values (VEVs) of GUT Higgs fields, large representations are usually required and a potential enforcing the desired VEV has to be specified. Furthermore, solving the doublet-triplet splitting problem without fine tuning adds extra complexity to the models.

An interesting alternative is provided by the Hosotani mechanism [1], which can be implemented in higher-dimensional theories compactified on manifolds with non-trivial topology. In this case, the symmetry breaking can be ascribed to the VEV of a Wilson line wrapping a non-contractible loop in extra dimensions. Related geometrical mechanisms of gauge symmetry breaking are used in string-compactifications on Calabi-Yau manifolds [2] and can naturally lead to doublet-triplet splitting [3]. Interesting relations between extra-dimensional topology and gauge symmetry breaking also exist in field-theoretic settings with non-zero field-strength and no supersymmetry (see, e.g., [4]).

A serious problem of field-theoretic GUT models with gauge symmetry breaking by the Hosotani mechanism is the flatness of the classical potential of the Wilson line, which is protected from loop corrections by supersymmetry (SUSY). Thus, one usually encounters light adjoint fields (with mass of the order of the SUSY breaking scale) ruining precision gauge coupling unification. The situation is improved in 5-dimensional field-theoretic orbifold GUT models [5] (see [6] for the original stringy idea), where the gauge symmetry is broken by boundary conditions and no adjoint moduli arise. Although related 6d GUT models [7] again have Wilson line VEVs, these lines can be contracted to zero length at conical singularities of the compact space, so that the VEV is fixed by a symmetry-breaking boundary condition. The presence of such boundaries (locations with reduced gauge symmetry) restricts the predictivity of orbifold GUTs because fields and operators that do not respect the GUT symmetry can be added at these points. It also introduces UV-sensitive corrections to gauge coupling unification, the natural size of which corresponds roughly to the thresholds of conventional 4d GUTs.

In this letter, an alternative field-theoretic mechanism for gauge symmetry breaking is considered. To understand the basic idea, it is sufficient to consider a 6d field theory with gauge group G compactified on a 2-sphere. Modding out by a Z_2 symmetry which acts on the sphere as a reflection with respect to the center and in gauge space by the inner automorphism $g \to PgP^{-1}$ (with $P \in G$, $P^2 = 1$), one obtains a gauge theory on the projective plane. The symmetry of the 4d effective field theory is reduced to the subgroup commuting with P. Alternatively, the model can be characterized as a sphere with the insertion of a crosscap (for the basic geometric concepts see, e.g., [8]), where the identification of the opposite edges at the crosscap is associated with a gauge twist P. It is easy to observe (see also below) that the topology requires the gauge twist to obey $P^2 = 1$. Thus, although the breaking is entirely non-local, no Wilson line modulus appears. More generally, this type of discretized topological breaking occurs in situations where the fundamental group of the compact space is non-trival but finite, such as in many Calabi-Yau models (see [2, 3] and, in particular, [9]). In this context, the real projective plane has been mentioned in [10]. The present realization combines the features of an extremely simple compact space with unbroken $N = 1$ SUSY and thus has all the ingredients necessary for realistic model building.

The paper is organized as follows: in section 2, it is shown how the above illustrative example can be promoted to a more interesting GUT-like model. In particular, starting from a 6d $N = 2$ super Yang-Mills (SYM) theory, a model with unbroken 4d $N = 1$ SUSY, broken gauge symmetry and no moduli is constructed by orbifolding. The geometry is such that the curvature of the topological 2-sphere discussed above is concentrated at four conical singularities. The construction involves modding out a freely acting discrete symmetry (cf. the freely acting orbifold models familiar in string theory, especially in the context of SUSY breaking [11]).

In section 3, an SU(6) model with doublet-triplet splitting is discussed. Given that the smallest truly unified group is SU(5), it is desirable that no SU(5) breaking fixed points exist. Doublet-triplet splitting is then most naturally realized if the Higgs is a bulk field. Since no gauged bulk matter is allowed by 6d $N = 2$ SUSY, the gauge group has to be extended to allow Higgs doublets to emerge from the adjoint representation, the minimal choice being SU(6). Indeed, it is possible to construct a model where SU(6) is broken to SU(5) at one of the fixed points (the further breaking being topological) and two naturally light Higgs doublets appear in a way closely related to the models of [12]–[14].

Section 4 is devoted to the fixed-point breaking of SU(6) to SU(5). For this several possibilities exist, the simplest one being to declare the gauge symmetry at the conical singularity to be reduced and to supply appropriate boundary conditions for the bulk fields. A more interesting possibility involves cutting off the tip of the cone by a gauge-symmetry-breaking 5d boundary and letting the length of the boundary tend to zero.

Conclusions and open questions are discussed in section 5.

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2. Non-locally broken SYM theory without moduli

Consider a 6d SYM theory with (1,1) SUSY and gauge group G, which can be thought of as deriving from a 10d SYM theory by torus compactification. (The large amount of supersymmetry is required to ensure the phenomenologically desirable $N = 1$ SUSY of the 4d theory obtained after orbifolding.) It will prove convenient to describe this theory in terms of a 4d vector superfield V and 3 chiral superfields with scalar components $\Phi_5 = A_5 + iA_8$, $\Phi_6 = A_6 + iA_9$ and $\Phi_7 = A_7 + iA_{10}$ [15]. Here A_5 and A_6 are the extra-dimensional (with respect to 4d) gauge field components of the 6d theory and $A_7 \dots A_{10}$ are 6d adjoint scalars deriving in an obvious way from the gauge field of the associated 10d SYM theory.

The 6d theory is compactified on a torus T^2 parameterized by $(x_5, x_6) \in \mathbb{R}^2$ with the identifications $x_5 \sim x_5 + 2\pi R_5$ and $x_6 \sim x_6 + 2\pi R_6$. In the first step of orbifolding, the rotation symmetry $(x_5, x_6) \to -(x_5, x_6)$ is 'modded out'. The action of this Z_2 symmetry in field space is chosen (using the symbol Φ_i for both the superfield and its scalar component) as $(\Phi_5, \Phi_6) \to -(\Phi_5, \Phi_6)$, with V and Φ_7 being inert. The fundamental compact space now has the topology of a 2-sphere with the curvature being concentrated at 4 conical singularities with deficit angle π and can be visualized as the surface of a 'pillow' [16]. The gauge symmetry is unrestricted.

In the second, crucial step, a freely-acting Z_2' symmetry is modded out, at which point the gauge symmetry breaking is introduced. The geometric Z_2' action is defined by $x_5 - \pi R_5/2 \to -(x_5 - \pi R_5/2)$ (reflection with respect to the line $x_5 = \pi R_5/2$) and $x_6 \to x_6 + \pi R_6$ (translation along that line). The action in field space makes use of the gauge twist $P \in G$ ($P^2 = 1$) and is given by $V \to PVP^{-1}$, $\Phi_5 \to -P\Phi_5 P^{-1}$, $\Phi_6 \to P\Phi_6 P^{-1}$ and $\Phi_7 \to -P\Phi_7 P^{-1}$ (cf. the rectangular models of [12]). The topology of the resulting fundamental compact space (which is equivalent to that of the projective plane) is illustrated in figure 1. The space is non-orientable, has no boundaries and the curvature is concentrated at the two fixed points F_1 and F_2, where conical singularities with deficit angle π reside.

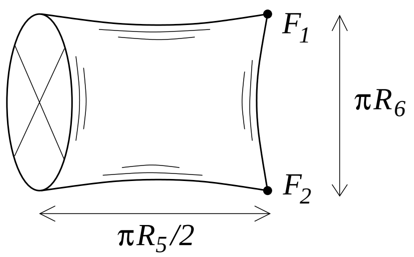

Figure 1: Illustration of the topology of the $T^2/(\mathbb{Z}_2 \times \mathbb{Z}_2')$ model discussed in the text. The manifold is flat everywhere except for the two conical singularities at F_1 and F_2. At the crosscap (symbolized by a circle with a cross) opposite points are identified, making the surface non-orientable.

Given the above Z_2 and Z_2' action, it is clear that the massless modes are the fields from V commuting with P (corresponding to the unbroken subgroup $H \subset G$) and the fields from Φ_7 anticommuting with P (corresponding to the complement of H in G). The latter ones can become heavy due to a mass term for Φ_7 localized at one of the fixed points. One 4d SUSY survives since the non-gauge part of both the Z_2 and Z_2' action on the Φ_i fits into SU(3) [2].

Thus, in the specific case $G = \mathrm{SU}(5)$ and $P = \mathrm{diag}(1, 1, 1, -1, -1)$ (cf. [5]), the gauge symmetry is broken to the standard model (SM) group in an entirely non-local fashion (the fixed points are SU(5) symmetric) and only the SM gauge multiplet is naturally light.

Although SM matter can be added at the two fixed points, it appears difficult to realize doublet-triplet splitting given the extremely soft nature of the breaking. By construction, the fixed points have 4d $N = 2$ SUSY which can, however, be broken simply by allowing for operators or a field content that are consistent only with $N = 1$.

The most interesting feature of the above toy model GUT is the absence of the Wilson line moduli that typically accompany a purely topological gauge symmetry breaking. This effect is linked to the small fundamental group of the projective plane ($\mathbb{Z}_2$ rather than, say, Z for an S^1) and can be understood as follows: By construction, a closed Wilson line loop passing once through the crosscap has the value P. A Wilson loop passing through the crosscap twice has the value P^2 and is, at the same time, contractible by the topology of the projective plane. Thus $P^2 = 1$ is required by consistency as long as the field strength vanishes everywhere including the fixed points (i.e., an infinitesimal loop surrounding a fixed point has value 1). In field theory, this latter statement corresponds simply to a gauge-invariant boundary condition at the singularity. The fact that such a boundary condition can not be continuously deformed is natural from the point of view of string theory (cf. the discrete or quantized Wilson lines of [6, 17]). Changing the value of the non-contractible Wilson loop while its square is fixed to equal unity corresponds to a change by inner automorphism and thus to an unphysical, gauge degree of freedom.

Note that it is possible to give the model an equivalent formulation with trivial (from the gauge theory perspective) transition function at the crosscap and non-vanishing, smooth gauge connection throughout the compact space. To see this, consider the same geometry as above and introduce a gauge field which, near the crosscap boundary, is pointed parallel to that boundary and has the same value everywhere along that boundary. This value can be chosen such that a Wilson line loop through the crosscap equals P. Furthermore, let the field strength be zero everywhere in the bulk and let the infinitesimal Wilson loop surrounding F_2 vanish. Then, by global topology (cf. the discussion of 'conifold GUTs' in [18]) the infinitesimal Wilson loop surrounding F_1 equals $P^2 = 1$ and thus gauge symmetry is unbroken at that point.

3. An SU(6) model with doublet-triplet splitting

As far as realistic model building is concerned, the most unsatisfactory feature of the above construction is the absence of doublet-triplet splitting. This problem can be overcome by extending the bulk gauge symmetry to SU(6) (cf. [12]–[14]) and using the gauge twist $P = \mathrm{diag}(1, 1, 1, 1, -1, -1)$ for the breaking at the crosscap. In addition, SU(6) is broken to SU(5) (defined as the submatrix containing the last 5 rows and last 5 columns of the original 6×6 matrix) at the fixed point F_1. For the moment, the precise mechanism of this breaking can be left unspecified. (One may imagine boundary localized Higgs fields or a hard, ad-hoc breaking by boundary conditions.) It is, however, important that no non-trivial Wilson line surrounding F_1 is introduced and that the gauge fields in V outside the SU(5) subgroup acquire mass due to local physics at F_1.

Now, the low-energy gauge symmetry is that of the SM and the only potentially light fields (in addition to the gauge fields) are part of Φ_7 and transform as

$$(\mathbf{3},\mathbf{2})_{-5}+(\bar{\mathbf{3}},\mathbf{2})_{5}+(\mathbf{1},\mathbf{2})_{-3}+(\mathbf{1},\mathbf{2})_{3} \tag{3.1}$$

under the SU(3) $\times$ SU(2) $\times$ U(1) subgroup of SU(5). They are associated with SU(5) multiplets relevant for the local physics at F_1:

$$(\mathbf{3},\mathbf{2})_{5}+(\bar{\mathbf{3}},\mathbf{2})_{-5}\subset\mathbf{24}\quad,\qquad(\mathbf{1},\mathbf{2})_{3}\subset\mathbf{5}\quad,\qquad(\mathbf{1},\mathbf{2})_{-3}\subset\bar{\mathbf{5}}\,. \tag{3.2}$$

Phenomenologically, the fields with the quantum numbers of X,Y gauge bosons have to become massive while the two pure doublets may stay light and play the role of the two SM Higgs fields. It is clearly possible to introduce extra symmetries at F_1 allowing the first $(\mathbf{24}\times\mathbf{24})$ and forbidding the second $(\mathbf{5}\times\bar{\mathbf{5}})$ SU(5) invariant mass term. SM matter could than be added in the form of brane-localized fields at F_1. The bulk should not be too large so that non-local effects can lead to a sufficient violation of the SU(5) symmetry of the F_1-based Yukawa couplings. Except for the somewhat ad hoc (though not fine-tuned) lightness of the Higgs doublets, the model now appears to be satisfactory.

However, it turns out that the above analysis is incomplete and that the local breaking of SU(6) to SU(5) at F_1 leads to the appearance of extra light states, not visible in the usual Kaluza-Klein mode expansion. The point is that some of the degrees of freedom associated with the Wilson loop going through the crosscap cease to be gauge artifacts and become physical fields. To be more specific, let the value of the closed Wilson loop beginning at F_1 and going once through the crosscap be $W\in$ SU(6). So far, we have assumed $W=P$. It will now be shown that changing W by inner automorphism (e.g., to $W'=UPU^{-1}$ with $U\in$ SU(6)) corresponds to the excitation of a physical degree of freedom if the gauge symmetry at F_1 is reduced. Before doing so, let us argue that this change of W is indeed a flat direction in fields space. For this, it is sufficient to consider a smooth scalar function f on the fundamental space (cf. figure 1) such that $f\equiv 0$ in a neighbourhood of the crosscap and $f\equiv 1$ in a neighbourhood of F_1. Writing $U=\exp(T)$, it becomes clear that the gauge fields that would be introduced by a gauge transformation $\exp(fT)$ lead to the desired inner-automorphism change of W while corresponding, at the same time, to a flat direction in field space.

Now, it remains to be shown that some of the freedom of rotating $W=P$ to $W'=UPU^{-1}$ corresponds indeed to physical fields. Group-theoretically, the SU(6) breaking at F_1 may be thought of as coming from a Z_2 twist $P'=\mathrm{diag}(-i,i,i,i,i,i)$. Given the restricted gauge symmetry at F_1, it is possible to write down the gauge invariant (and thus observable) operator $\mathrm{tr}(W'P')=\mathrm{tr}(UPU^{-1}P')$. Clearly, excitations generated by Lie algebra elements T that anticommute with both P and P' lead to a change of the value of this operator. Their quantum numbers are the same as those of the two SM Higgs doublets obtained above from the chiral superfield Φ_7. However, their physical interpretation is entirely different. While the former are conventional bulk zero modes with potential brane-localized mass terms, the latter are non-local degrees of freedom parameterizing the relative orientation of the two breaking patterns P and P'. Thus, no local mass term or couplings to other fields can be written down. However, such couplings may be generated non-perturbatively or by integrating out appropriate gauged bulk fields [10, 19].

There are now various options for solving the doublet-triplet splitting problem: One can use the Higgs doublets from Φ_7 and argue that effective non-local operators will make the Wilson-line doublets heavy. This means that the problem of Wilson-line moduli (avoided by the crosscap breaking) is partially reintroduced and then solved in an ad-hoc way. Thus, it appears more natural to give all Φ_7 fields brane masses and to view the light Wilson line doublets, the existence of which is deeply rooted in the geometry of the model, as natural candidates for light Higgs fields. With matter localized at F_1, Yukawa couplings now arise from non-local effects. The main potential problem of this scenario is the difficulty of generating a sufficiently large top mass. Finally, the situation could be more involved and the required two light Higgs fields might arise as a non-trivial linear superposition of the four potentially light doublets. For the purpose of this paper, it is sufficient that the existence of natural solutions to the doublet-triplet splitting problem has been demonstrated. The analysis of possibilities for generating realistic Yukawa couplings is postponed to a further investigation.

4. Breaking SU(6) to SU(5) at a singularity

In the previous section, it was simply assumed that the gauge symmetry at F_1 is reduced to SU(5). Here, the corresponding possibilities for symmetry breaking are analysed in more detail and, in particular, a realization within the framework of field-theoretic orbifolding is suggested.

Clearly, one of the options is symmetry breaking by a brane localized Higgs [20]. In this case, it is important to use a set of VEVs and appropriate extra symmetries ensuring that the **24** from Φ_7 acquires a brane mass while the **5** and $\bar{\mathbf{5}}$ from Φ_7 remain light.

Within the framework of field theoretic orbifolding, the most natural choice appears to be a Wilson line with value P' wrapping F_1. However, introducing such a Wilson line gives mass both to the doublets from Φ_7 as well as to the Wilson line doublets discussed in detail in the previous section. Thus, doublet-triplet splitting becomes a serious problem.

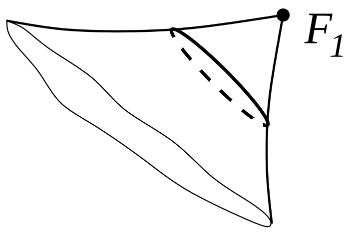

Figure 2: Illustration of a possible resolution of the singularity at F_1. On taking the length of the cut to zero (i.e., cutting out a very small neighbourhood of F_1), the global structure of the original model of figure 1 is recovered.

Therefore, this section focusses on an alternative option where, as illustrated in figure 2, the singularity F_1 is cut out and removed from the fundamental space. The group SU(6) is broken by the Z_2 reflection used to define the boundary conditions at the 4+1 dimensional brane created by this cut (cf. the disc and annulus models of [21]). After cutting out F_1, the global topology is that of a Möbius strip. One might be concerned that now moduli (corresponding to Wilson lines wrapping the cycle of the Möbius strip) appear after all. However, they can be given a mass by appropriate non-local operators involving, e.g., the Wilson loop along the boundary. Taking the limit where the length of the boundary tends to zero, such operators become effectively local.

To be more specific about the boundary conditions, consider first a small interval on the boundary (implying that the curvature can be neglected) at a point (x_0^5, x_0^6) where the boundary is parallel to the x^6 direction. One can then think of this boundary locally as being defined by the reflection $x^5 - x_0^5 \to -(x^5 - x_0^5)$ together with an action in field space $V \to P'VP'^{-1}$, $\Phi_5 \to -P'\Phi_5 P'^{-1}$, $\Phi_6 \to P'\Phi_6 P'^{-1}$ and $\Phi_7 \to -P'\Phi_7 P'^{-1}$. Choosing the boundary in the shape of a circle on the original torus, SO(2) rotation symmetry acting on the superfields (Φ_5, Φ_6) as well as in coordinate space is now used to fix the boundary conditions locally at every point along the cut. In particular, although different linear combinations of Φ_5 and Φ_6 are forced to vanish as one moves along the boundary, the fields from Φ_7 anticommuting with P' are always allowed to be non-zero. Thus, in the complete model, the Φ_7 chiral superfield contributes precisely two potential Higgs doublets to the low-energy field content. This was realized at the beginning of the previous section by ad-hoc assumptions about local symmetry breaking and mass terms localized at F_1. Obviously, the breaking by P' leads to a surviving U(1) in addition to SU(5). However, it does not affect the phenomenology at the present rough level of discussion and different options for the scale and mechanism of its breaking may be considered.

The above discussion of the $\mathbb{Z}_2$ boundary condition ignored the curvature of the boundary in the flat 2d extra-dimensional space and treated it as a sum of straight elements. To describe the actual curved boundary, one can simply go by diffeomorphism to a coordinate system where the boundary is straight (and a corresponding non-zero Riemann connection, which is considered non-dynamical, appears). Now a finite piece of the boundary can be defined by Z_2 reflection. In addition to the Riemann connection, an extra non-zero, non-dynamical R-symmetry connection has to be explicitly introduced in this picture. It accounts for the appropriate rotation of A_8, A_9 and the fermions in the Φ_5 and Φ_6 superfields as one moves along the boundary.

5. Conclusions

In this paper, gauge symmetry breaking by a specific topological feature, namely a crosscap, of the extra-dimensional manifold was considered. The main attributes of this simple mechanism are non-locality (implying the 'softness' of the breaking), absence of flat directions (in particular Wilson line moduli), and the ease with which it can be implemented in a higher-dimensional SUSY GUT framework.

If, in addition to the crosscap breaking, the gauge symmetry is broken by a different mechanism at some point in extra-dimensional space, Wilson-line moduli (characterizing the relative orientation of the two breakings) reappear. For a breaking pattern leading from SU(6) to SU(5) and further to the standard model, this implies the existence of two light Higgs doublets.

In the context of the above SU(6) model, the main open questions concern the generation of Yukawa couplings and the possible role of two further potentially light Higgs doublets not associated with Wilson lines. In a wider context, it would be interesting to consider larger groups, to attempt to generate matter and Yukawa couplings from the pure SYM theory [14, 22], and to use the softness of the breaking to perform a calculation of GUT threshold corrections. Furthermore, a systematic understanding of other geometries (possibly in more than 2 extra dimensions) allowing for this type of non-local but quantized gauge symmetry breaking would be desirable.

Acknowledgments

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